53. Reversibility + Irreversibility.

Let us return, for the moment, to the probabilistic interpretation of quantum theory, based upon theorems 1 and 2 (and which is equivalent from the viewpoint of a subjective level, as we have seen). Let us consider measurements as a means of converting states into mixtures, namely, that measurement of a non-degenerate operator \( A \), with eigenstates \( \{ \phi_j \} \), converts the state \( \psi \) into the mixture of \( \phi_j \) weighted by \( \beta_j = |(\psi, \phi_j)|^2 \), i.e. the mixture whose density operator is \( \rho = \sum_j |(\psi, \phi_j)|^2 \phi_j \phi_j^\dagger \).

We might, for example, suppose that the measurement was performed by a suitable apparatus, but that we did not learn the value recorded by the apparatus -- or that another observer performed the observation. In such a case we can no longer...
assign a state, but only a probability mixture. We might simply take the view that we originally had a large ensemble of systems, and that the measurement was performed upon each member, so that as a result the ensemble would consist of states in the set \( \Phi_i \), in the proportions given by \( \frac{1}{i} (|\psi_i\rangle \langle \psi_i|) \).

More generally, we might start with a mixture, \( \tilde{\Phi} = \sum_i p_i |\psi_i\rangle \langle \psi_i| \), in which case the measurement of \( A \) produces the new mixture \( \tilde{\Phi}' \), on which we get the rule easily by summing the individual terms:

\[
\tilde{\Phi}' = \sum_i \tilde{p}_i \sum_j \left| (\tilde{\psi}_j, \phi_i) \right|^2 |\phi_i\rangle \\
= \sum_i \left( \sum_j \frac{\tilde{p}_j}{\tilde{p}_i} (\langle \tilde{\psi}_j | \langle \psi_i | \psi_i \rangle \langle \psi_i | \tilde{\psi}_j \rangle) \right) |\phi_i\rangle \\
= \sum_i \left( \phi_i \sum_j \frac{\tilde{p}_j}{\tilde{p}_i} |\psi_i\rangle \langle \psi_i | \phi_j \rangle \right) |\phi_i\rangle \\
= \sum_j \left( \frac{\tilde{p}_j}{\tilde{p}_i} \sum_i \phi_i |\psi_i\rangle \langle \psi_i | \phi_j \rangle \right) |\phi_j\rangle
definition
For each mixture \( \rho \), we define a quantity \( I_\rho \):

\[
I_\rho = \text{Trace} (\rho \ln \rho)
\]

(3.9)

This number, \( I_\rho \), has the character of an information ordinate, in fact, the information of the probability distribution over the eigenstates of \( \rho \) (i.e., \( \rho \) can always be brought into the form \( \rho = \sum_i \rho_i \left| \phi_i \right\rangle \left\langle \phi_i \right| \), where the set \( \{ \phi_i \} \) is orthonormal.

Then \( I_\rho = \text{Trace} \rho \ln \rho = -\sum_i \rho \ln \rho \), which is simply the information in the distribution over the states \( \{ \phi_i \} \) of the mixture. It is also (see ) the negative of the dimensionless entropy of \( \rho \).

Process 2 has the property that it leaves \( I_\rho \) unchanged, since:

\[
I_\rho' = \text{Trace} (\rho' \ln \rho') = \text{Trace} \left( \sum_i \rho_i \left| \phi'_i \right\rangle \left\langle \phi'_i \right| \ln \sum_i \rho_i \left| \phi'_i \right\rangle \left\langle \phi'_i \right| \right)
= \text{Trace} (\rho \ln \rho) = I_\rho.
\]

Process 1, on the other hand, can decrease \( I_\rho \), but never increase it, since, by (3.1):

\[
\rho' = \sum_i (\rho_i \left| \phi'_i \right\rangle \left\langle \phi'_i \right|) = \sum_i \rho_i \left( \frac{\left| \phi'_i \right\rangle \left\langle \phi'_i \right|}{\rho_i} \right)^2 \left| \phi'_i \right\rangle \left\langle \phi'_i \right|
= \sum_i \rho'_i \left| \phi'_i \right\rangle \left\langle \phi'_i \right|
\]

where \( \rho'_i = \sum_i T_{i'i} \) and \( T_{i'i} = \left( \frac{\left| \phi'_i \right\rangle \left\langle \phi'_i \right|}{\rho_i} \right)^2 \) is a doubly stochastic matrix. But \( I_\rho' = \sum_i \rho'_i \ln \rho'_i \) and \( I_\rho = \sum_i \rho \ln \rho_i \) with the \( \rho_i \) connected by \( T_{i'i} \), implies by the theorem of information decrease of stochastic process 2 that:

\[
I_\rho' \leq I_\rho.
\]
(It will, in fact, strictly decrease unless (for all $p_j > 0$) $T_i = 1$ for some $i$ and 0 for the rest ($\frac{\delta}{\xi^2}$)

which means $\sum_j (\frac{\delta_{ij}}{\xi_j})^2 \xi_j = \phi_k \Rightarrow \phi_j = \phi_k$ for some $k$.

for every $j$ for which $p_j > 0$ —— which means that the original mixture was already a mixture of eigenstates of the measurement.)

Therefore, it is not possible to get from

any mixture $\rho$ to any other one $\sigma$ means of processes 1 and 2. There is an essential irreversibility in process 1, which cannot be compensated by process 2.

Puzzle: How our theory must give equivalent result on the subjective level? It indicates another way of looking at the irreversibility. There is another way of looking at this irreversibility, within our theory which recognizes only process 2. When an observer performs an observation, we have as a result a superposition, each element of which describes an observer who has perceived a particular value. From this time on, there is no interaction between the separate elements of the superposition, since each element separately obeys the wave equation — i.e., starting the observer, each observer described by a separate element behaves completely independently of any of remaining elements. From this viewpoint, each such observer is permanently cut off from the remainder of the elements — he cannot longer obtain any information whatsoever about them (they are completely knowable to him ——). Therefore, the irreversible phenomena in nature are, within our framework, again simply subjective manifestations —— conceivable that some outside entity could reverse whole W.T.
transition probabilities

usually done for one system $\hat{S}$, Hamiltonian $H$
stationary states $\psi_i$

Then perturbation introduced $H_\nu(x)$ (as a charge $H$, i.e. $H \mapsto H + H_\nu$)

under $H_\nu$, $\phi_i \rightarrow \psi_i(x) = \sum_j (\phi_j , \psi_i(x)) \phi_j$

i.e. the state $\phi_i$ transformed into superposition

and the coefficients $\alpha_i^{(\nu)} = (\phi_i , \psi_i(x)) \phi_j$

give the time dependence.
By definite this measurement (with n eigenvalues $\lambda_i$)
and as initial wavepacket (spatial origin $\Psi_0$) is made
the probability distribution (in configuration) for the result is

$$\rho_0 = \sum_i \lambda_i |\Psi_i(0)|^2$$

and the quantity $|\Psi_i(t)|^2$ are called transition
probabilities and the limit

\[ \lim_{t \to \infty} |\Psi_i(t)|^2 \]

In this case, however, the true transition probability
is somewhat a phenomenon, since it contains the
deduction that the initial state $\Psi$ is transformed into
a mixture, of the $\Psi_i$ with weights $\rho_i$. This is incorrect,
since there is still a pure state, $\Psi = \sum_i \lambda_i \Psi_i$, and the
interference phenomena are still present between the states $\Psi_i$. The
operators differ from the energy eigenvalues, so their expectation
must be calculated from the superposition, and not the matrix.
Apprec. Meas.

Emphasizing difficulty of trying to fit into excluded from ol Proc I 2.
After two systems have interacted and become correlated, it is true that marginal expectation for subsystems operators can be calculated correctly when the composite system is represented by a non-interfering mixture of states, which is the representation must be regarded as only a mathematical fiction useful in many cases, yet which is true if the composite system state is \( \psi_{\text{sys}} = \sum_{i} a_{i} \phi_{i}^{\text{sys}} \), where the \( \{ \phi_{i}^{\text{sys}} \} \) are orthogonal.

Thus, for purposes of calculating the expectations of operators on \( \mathcal{S}_{\text{sys}} \), \( \psi_{\text{sys}} \) is equivalent to the non-interfering mixture of states \( \sum_{i} a_{i}^{*} \phi_{i}^{\text{sys}} \) weighted by \( a_{i}^{*} a_{i} \), and one can picture that each another of the cases \( \phi_{i}^{\text{sys}} \) has been realized to the exclusion of the rest, with probabilities \( a_{i}^{2} \). However, this representation by mixtures must be regarded as only a mathematical artifact, which, although useful in many cases, is an incomplete description because it ignores phase relations which actually exist between the separate elements, which become important in many interactions which involve the system (indistinguishably to just a subsystem, more than just a subsystem). (In the present example, the phase relations of the two beams, corresponding to separate elements of a superposition of the two beams, and 3-component apparatus are regarded as, for example the "composite system" is separate of the subsystems' spin value (object system) and 3-coordinate (apparatus) and the superposition is the two diverging wave packets...)}
We take this opportunity to caution against a certain viewpoint which can lead to difficulties. This is the view that when an apparatus has interacted with a system, that in "actually" one or another of the elements of the resultant superposition described by the state function has been realized to the exclusion of the rest, the existing one simply being unknown to an external observer. This view must be erroneous, since there is always the possibility for the external observer to make use of interference properties of the superposition. An example is cited by Bohm of a Stern-Gerlach apparatus which separates an incoming beam of atoms, initially in 1\,\text{by eigenstates}, into two specially
is separated beams corresponding to the two eigenstates of $\lambda$. He then illustrates the fact that it is in principle possible to deflect the two beams (with magnets) back into one beam, preserving the phase relationships, so that the state is again the original eigenstate of $\lambda$. In this case it would be erroneous to hold the view that the Stern-Gerlach apparatus has caused the atom to be definitely in one or the other of the $\lambda$ eigenstates and hence in one or the other beam. The view that the Stern-Gerlach apparatus behaved in accordance with process 1 to convert the state into a non-interfering mixture of eigenstates -- since the possibility of combining the beams and making use of the interference properties is still open (thus both beams must be regarded as existing in the state "real"). In general it is not proper
to attribute any validity, or "reality," to any element of a superposition than any other element, due to the ever present possibility of obtaining interference effects between the elements.

Bell's discussion of complexity of macroscopic object (many degrees of freedom) causing (essentially) system state change into non-interfering mixture

is actually simply a statement that (in system eigenstate representation, relative apparative states are orthogonal to on extremely high degree of approximation, (the non-interference between superposition elements in this representation, for operators on system only, is then ensured by the orthogonality of the apparative state also, (this representation is then "near" the canonical representation of Chap IV)
This thus in this sense of correlations between constituent particles that definite macroscopic objects exist within the present theory. The wave function for the centroid can therefore be taken as a representative wave function for the whole object.

Include:

a) condensation phenomena (cf. to Shroedinger)
b) refers to the "memory state" of ideal Weaver

classical mechanics

Thus observers since

Wave functions can describe classically functioning

mechanics - from which we can form the

automate representation of our observers.